J. Finan. Intermediation 17 (2008) 444–463

Contents lists available at ScienceDirect

J. Finan. Intermediation

www.elsevier.com/locate/jfi

Credit derivatives, capital requirements and opaque OTC markets

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ARTICLE INFO

Article history:
Received 25 February 2006
Available online 18 April 2008

JEL classification:
G21
D82

Keywords:
Credit derivatives
Signaling contracts
Capital requirements

ABSTRACT

In this paper we study the optimal design of credit derivative contracts when banks have private information about their ability in the loan market and are subject to capital requirements. First, we prove that when banks are subject to a maximum loss capital requirement the optimal signaling contract is a binary credit default basket. Second, we show that if credit derivative markets are opaque then banks cannot commit to terminal-date risk exposure, and therefore the optimal signaling contract is more costly. The above results allow us to discuss the potential implications of different capital adequacy rules for the credit derivative markets.

1. Introduction

This paper investigates the optimal design of credit derivative contracts when banks have private information about their loan portfolios and are subject to capital requirements. The need for this study comes from two main events that have characterized the last decade.

First, in most jurisdictions around the world bank capital regulations are changing from Basel I[1] to Basel II. While in the former capital requirements depend on asset risk buckets and credit derivatives are not considered, in the latter capital requirements are based on maximum losses due to loan default.[2]

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[1] See BCBS (1988).

[2] This is the Value at Risk approach. See BCBS (2005).

1042-9573/$ – see front matter
doi:10.1016/j.jfi.2008.03.001

Second, the recent advent of credit derivatives has provided banks with a range of flexible instruments for selling loans and transferring loan risk. For example, pure credit derivatives, such as plain vanilla credit default swaps (CDSs), allow banks to buy protection on a single exposure or on a basket of exposures. Similarly, portfolio products, such as Collateralized Debt Obligations[3] (CDOs), enable banks to sell risks from their entire loan portfolio.[4] One main advantage of these new instruments over the traditional forms of credit risk transfer is their flexibility, which helps mitigate informational problems. Banks differ in their abilities to screen their loans, which are unobservable by the credit risk-buyers. Consequently, they face an asymmetric information problem when they want to sell risks from their portfolios. Banks with superior abilities design credit derivative contracts to signal their quality in order to overcome the adverse selection problem generated by the asymmetry of information. However, as the recent sub-prime crisis has highlighted, the growth in volume and diversity of credit derivative products has not mitigated the problem of the lack of transparency in such markets.[5] Market opacity may affect the design of credit derivative contracts since it prevents banks from credibly committing to retain part of the risk of their portfolios.

The debate over the sub-prime crises raises two policy-relevant research questions that have not yet been considered in the literature. First, how do the different institutional settings—namely, the introduction of new regulatory capital requirements based on maximum loss—influence the optimal design of contracts that banks may offer to signal their own types? More specifically, is the equity tranche holding, largely used by banks in recent years, an optimal signaling contract under the new Basel II regulatory framework? Second, how does the opacity in the credit derivative markets affect the design of the credit derivative contracts? Our analysis allows to answer both questions by determining the optimal security design.

We show that the optimal signaling contract when banks are subject to maximum loss capital requirement is a binary credit default basket (Binary CDB), i.e., a credit default basket where the payoff is a fixed amount that can be considered a penalty payment when defaults are above a certain level. Accepting exposure to loss is a credible signal that a bank's probability of loss is low. When banks can commit to loss exposure and capital requirements impose a dissipative cost of exposure based only on maximum losses, the cheapest way for banks to signal their superior ability is to increase exposures while minimizing the maximum losses. This leads to equal exposures across states and thus to the binary CDB contract. In contrast, if a bank cannot commit to terminal-date risk exposure because of opacity, it must signal its superior ability with its initial-date price. However, since a high-type bank does not have any competitive advantage over low-type banks in burning its money with underpricing, such signaling is very costly, much more than signaling under transparency.

The above results allow us to investigate the potential implications of different capital adequacy rules for the optimal separating contracts, and whether existing contracts are consistent with our predictions. We show that the optimal contracts are largely dependent on bank regulation and argue that in the future we will observe an evolution of different credit market instruments. The introduction of Basel II may prevent the use of the equity tranche as a signaling device, as it requires all first loss positions to be deducted from bank capital. We suggest that, under mild conditions, contracts like Binary CDB are optimal. However, the opaqueness in the credit derivative market could make the use of these signaling contracts quite expensive. The recent implementation of Basel II does not allow us to empirically test predictions of our model. Nevertheless, we investigate some credit market innovations that are in line with them, such as the BISTRO product and credit derivative product companies.

The paper is organized as follows. The next section describes the related literature. In Section 3 we present the basic model and we analyze the benchmark case with symmetric information. In

[3] A CDO is an asset-backed security whose underlying collateral is typically a portfolio of bonds or bank loans. A CDO cash-flow structure allocates interest income and principal repayments from a collateral pool of different debt instruments to a prioritized collection of CDO securities, usually called tranches. A standard prioritization scheme is known as simple subordination: senior CDO tranches are paid before mezzanine and lower-subordinated notes, with any residual cash flow paid to an equity tranche.

[4] Surveys of BBA (2002, 2006), BIS (2003, 2005) and Minton et al. (2006) show that the volume of trade in credit derivatives has seen a huge increase in recent years.

[5] For the role of transparency in credit derivative markets during the recent sub-prime crisis, see BIS (2007).

Section 4 we consider the asymmetric information case and present our results. In Section 5, we investigate some credit market innovations that are in line with the predictions of our model. Section 6 concludes.

2. Related literature

The tremendous development in credit derivative markets has received the attention of both regulators and policy makers. Although most international and national supervisors have published reports on the topic (e.g. IMF, 2002; BIS, 2003, 2005), theoretical work on these issues is still lacking. Exceptions are Duffee and Zhou (2001), Morrison (2005), and Parlour and Plantin (in press).

Duffee and Zhou (2001) demonstrate that the problem of adverse selection may be overcome by drawing up credit derivatives with a smaller maturity than that of the underlying asset.[6] The key assumption in their model is that the information advantage of a bank changes over time and is greater when it is close to the maturity date of the loan. Our approach differs, as it assumes neither that the bank's information advantage decreases over time, nor that there is perfect observability of credit derivative contracts.

Morrison (2005) shows that if credit derivative trades are opaque, and thus the protection buyer cannot make an ex ante commitment to a specific protection level, banks have a moral hazard incentive to fully hedge their exposure and therefore cease to monitor. This behavior has the negative effect of causing disintermediation, thereby reducing welfare. In the present work we take a different approach than Morrison (2005) by investigating the adverse selection problem and showing how the opacity in the credit derivative markets affects the design of the optimal credit derivative contracts.[7]

Parlour and Plantin (in press) consider the effect of credit risk transfer on relationship banking. They focus on the optimal mix of equity, bonds, and loans, and show that liquidity effects can arise endogenously in credit derivative markets when banks are net protection buyers. As for Duffee and Zhou (2001) and Morrison (2005), Parlour and Plantin (in press) focus on the CDS market and, in contrast with our paper, they do not address the security design issue.

Even ignoring the capital requirement issue and the contract observability problem, the theoretical literature on credit derivatives and the problem of asymmetries of information is limited. DeMarzo and Duffie (1999) and DeMarzo (2005) focus on liquidity problems with asymmetric information. More precisely, they show that pooling and sharing may be optimal when the protection buyer has superior information. Our work differs in two main aspects. First, by introducing opacity of the credit derivative market, we rule out the possibility of using the quantity of the security sold on the market as a signaling device. Second, we do not focus on frictions generated by expected liquidity costs, but on costs induced by capital requirements based on maximum losses due to loan default.

Finally, we avoid the limited liability constraints and allow the (re)payment due to the buyer of the security to be larger than the cash-flow of the asset. Thus, we allow for contracts, unfeasible in a setting like that of DeMarzo and Duffie (1999), in which the signaling contracts' cash flows are restricted to be lower than those of the underlying portfolio in all states.[8]

[6] Moreover, Duffee and Zhou (2001) show that the mechanism proposed by Gorton and Pennacchi (1995) to reduce the moral hazard problem associated with the loan sales is broadly applicable to any mechanism that transfers loan risk outside of the bank, including credit derivatives.

[7] Chiesa (2005) shows that Morrison's (2005) problem could be solved using an optimal credit risk transfer instrument, i.e. transferring exogenous risk. In this way the bank lowers the amount of capital that must be put at stake for finding an incentive-compatible mechanism to monitor/screen the loans it originates.

[8] For a more complete overview of the issues that arise with various instruments for credit risk transfer see Kiff et al. (2003). For studies that assume symmetric information and investigate risk-sharing effects, see Allen and Carletti (2006). Empirical evidence on asymmetries of information and insider trading in the CDS markets and on why banks use credit derivatives are provided respectively by Acharya and Johnson (2006) and Minton et al. (2006).

3. The model

3.1. Assumptions

Let us consider a market where there is a bank (risk-seller) operating in the local loan market. The bank may hedge its exposures in the credit derivative market by selling credit risk to other financial institutions (risk-buyers[9]).

Both bank and risk-buyer are risk-neutral and, for simplicity, the riskless interest rate is zero. The bank belongs to one of two different types: high type (denoted by h) and low type (denoted by l). Types only differ in the quality of their loan pools for the credit risk for which the bank seeks protection. The quality of the pools is assumed to depend on borrowers' ability to repay loans and therefore the bank's quality can be represented by the probability that its borrowers repay loans. This probability is greater for a high-type than for a low-type bank. Let p_i for $i = \{h, l\}$ be the probability of success for loan repayment; then $0 \leqslant p_l < p_h \leqslant 1$, where p_l and p_h are the probability of loan success held by a low-type and a high-type bank, respectively.

The model incorporates two dates: 0 and 1. On date 0, the bank holds a portfolio of two commercial loans with fixed size: I_1 and I_2, which mature on date 1. Both credit lines can default only at the maturity date and are uncorrelated.[10] We assume that the recovery value is equal to zero. Moreover, all the cash flows occur at the maturity of the contracts.

Making a loan of amount I, a bank $i = \{h, l\}$ obtains an expected profit $E\pi_i = p_i(1+\mu)I - I$, where μ is the interest rate, which is the same for both types. Hence, risk-buyers cannot infer banks' types from the interest rate.[11] We assume that $\mu \leqslant 1$; this assumption allows us to simplify our analysis and is sufficiently mild not to undermine the generality of our results. We also assume that $E\pi_h > E\pi_l \geqslant 0$, that is both types of loans have non-negative net present value (NPV). The loan portfolio cash flows S are characterized by four states; State 1: no default ($S_1 = (I_1 + I_2)(1 + \mu)$), and the three default states, State 2: default of only loan 1 ($S_2 = I_2(1+\mu)$), State 3: default of only loan 2 ($S_3 = I_1(1+\mu)$), State 4: default on both loans ($S_4 = 0$).

Banks finance the portfolio of loans with deposits. Therefore, losses will push a bank towards insolvency if it has insufficient capital to cover them. Because of capital requirements, a bank, even if risk-neutral, cares about risk and holds capital as a buffer to cover losses. We assume that a bank must satisfy capital requirements based on maximum loss. Bank's capital is invested in a short term asset and is not used to finance the portfolio of loans. The unitary cost of capital, denoted by ρ, is greater [12] than the cost of deposits, which is normalized to zero.[13]

These assumptions simplify the model considerably. In fact, we do not need to model explicitly the bank's capital structure nor its capital adequacy rules in detail. Instead, we refer more generally to the bank capital needed to prevent default, with a given probability α. For simplicity, we assume that $(1-\alpha) < (1-p_h)^2$. This implies that the bank capital has to cover all losses.[14]

We focus on the case in which banks use credit derivatives in order to reduce capital requirements and therefore the cost of capital. The credit derivative market we consider is characterized by the presence of different types of contracts. At time 0, the bank simultaneously offers credit derivative

[9] Risk-buyers can also be banks. However, to avoid confusion, we identify the risk-seller with the bank and we generically refer to the other party as the risk-buyer.

[10] The key aspects of the paper are not based on diversification opportunities; the assumption that there are no other assets in the bank's portfolio allows us to present our results in a very simple framework focusing on the use of credit derivatives to cover exposures.

[11] This assumption is in line with the statement of Duffee and Zhou (2001), according to which there is no one-to-one relation between the interest rate charged by a bank and the quality of its borrowers.

[12] See Dewatripont and Tirole (1993), Froot and Stein (1998) and Gorton and Winton (1998) for explicit models of why ρ might be positive.

[13] Introducing a positive flat deposit insurance premium would reduce the spread by the cost of capital but this extra cost will be reflected in the cost of loans without altering our results.

[14] We discuss a relaxation of this assumption in Section 4.3.

contracts to the risk-buyers.[15] Since there are many risk-buyers, we assume that the bank faces a competitive market. At the time of the proposal, the bank's type is private information. The bank has full bargaining power and makes a take-it-or-leave-it offer to a risk-buyer. All the cash flows (including payment of premiums) occur at the maturity of the contracts. Finally, let $q \in (0,1)$ be the ex ante probability that the bank is of high quality.

3.2. The benchmark case: symmetric information

If a bank is issuing a portfolio of loans (I_1, I_2), its maximum loss, with a confidence level α, is $I_1 + I_2$. Under this framework, in order to satisfy capital requirements the bank has to guarantee loan losses with bank capital equal to $I_1 + I_2$. Let $E\pi_i$ denote the expected profits of a bank of quality $i = \{h, l\}$ when it does not transfer its risk. We have:

$$E\pi_i = p_i(I_1 + I_2)(1+\mu) - (I_1 + I_2) - \rho(I_1 + I_2) \quad \text{with } i = h, l \tag{1}$$

where $\rho(I_1 + I_2)$ is the required remuneration for the bank capital buffer that guarantees loan losses. In order to reduce the capital buffer, the bank can sign credit derivatives. In our model, the credit event is identified with a failure to pay at the maturity date. The credit event payment is defined as the difference between the nominal value plus the accrued interest and the recovery value of the defaulted loan, which in our framework is equal to zero.

When the bank's type is common knowledge, the lowest premium that a risk-neutral risk-buyer is willing to accept is the fair premium. Hence, the lowest premium that will fully insure a bank of type i with loan I_j by means of a plain vanilla CDS contract is:

$$\Phi_i(I_j) = (1 - p_i)I_j(1+\mu) \quad \text{with } i = h, l \text{ and } j = 1, 2. \tag{2}$$

Signing a CDS basket contract on the bank's loan portfolio, the expected profits of a bank of type i, denoted $E\pi_i(CDS)$, are

$$E\pi_i(CDS) = \mu(I_1 + I_2) - \Phi_i(I_1) - \Phi_i(I_2) \quad \text{with } i = h, l. \tag{3}$$

Observe that loan losses are completely covered by the CDS basket and therefore there are no capital requirements and no costs of capital. Since by assumption the NPV of the loans is positive for both types $i = h, l$, it follows that expected profits are positive.

It is straightforward to note that, with complete information, the full coverage CDS basket is a first-best contract.

4. Asymmetric information

4.1. Pooling equilibria

In any pooling equilibrium the minimal premium that a risk-buyer is willing to accept in order to sign a plain vanilla contract that fully hedges the bank against the credit risk of the loan I_j is

$$\Omega(I_j) = q(1 - p_h)I_j(1+\mu) + (1-q)(1-p_l)I_j(1+\mu) \quad \text{with } j = 1, 2. \tag{4}$$

Signing a full coverage CDS basket, a bank of type i obtains the following expected profit:

$$E\pi_i(FCPL) = \mu I_j - \Omega(I_j) \quad \text{with } i = h, l \text{ and } j = 1, 2. \tag{5}$$

As usual, it is easy to find the pooling equilibrium where both types of banks sign the same contract. In particular, it is straightforward to check that there exists a pooling equilibrium such that banks of both types sign the CDS basket. The risk-buyers' beliefs are such that, if a full coverage CDS basket is offered, the bank is of high type with probability q; if any contract different than a full coverage CDS basket is offered, then the bank is a low-type with probability 1. Clearly, high-type banks' profits are lower than their profits in a game with complete information; moreover, the lower the number of high-type banks in the market, the stronger is the incentive to signal their own type.

[15] We assume that risk buyers are not subject to capital requirements because, in line with empirical evidence, they are largely insurance companies or hedge funds (see BIS, 2005).

4.2. Separating equilibria

In this section we prove the existence of separating equilibria and we show how the optimal signaling contract varies according to the presence of capital requirements and the banks' ability to commit to retain some risk. We restrict our attention to contracts according to which a bank, in order to satisfy capital requirements, sells its loans by creating a security. The payoff of the security, F, is made contingent on the outcome of the unique verifiable signal, the cash flows of the loans, i.e. $F(S)$. The design of the security may vary and we define the optimal security as that which maximizes the bank's profits. Let P be the price of the security.

4.2.1. Optimal separating contract with no cost of capital

In our framework, when capital is costless a bank has no incentive to create a security. However, the case of costless capital is a useful benchmark for analyzing how the presence of capital constraints and the opacity of the credit derivative market affect the optimal design of the security. Since both bank and risk-buyer are risk-neutral there exist different forms for transferring the risk. These are equivalent in terms of welfare and provide the first-best level of profits $(p_h(1+\mu)-1)(I_1+I_2)$ to the high-type bank. An optimal credit derivative is the trivial contract in which the bank sells its loans at price $P^{PS}=(I_1+I_2)(1+\mu)$, and commits to fully insure the risk-buyer (that is, to repay $I(1+\mu)$ in case of default of loan I). More formally, the bank sells its loan portfolio and writes a Put with strike price $K=(I_1+I_2)(1+\mu)$, i.e. $F(S)=\max(K-S,0)$ and the contract is $[P^{PS},\max(K-S,0)]$.

When capital is costly, it is also useful to compare another credit derivative contract, the binary credit default basket contract (Binary CDB), with the optimal separating contracts. According to this security, the issuer commits to pay a fixed dollar amount L to the risk-buyer in case of default of any loan, where the amount L is fixed and independent with respect to the loss suffered by the risk-buyer, i.e. $F(S)=(L|S<(I_1+I_2)(1+\mu))$. The optimal credit derivative contract $[P^{NC},L^{NC}]$ is such that

$$P^{NC}=(I_1+I_2)(1+\mu)\left(\frac{1+p_hp_l}{p_h+p_l}\right) \tag{6}$$

and

$$L^{NC}=\frac{(I_1+I_2)(1+\mu)}{(p_h+p_l)}. \tag{7}$$

The following proposition summarizes the above discussion:

Proposition 1. *When bank capital is costless, there exists a separating equilibrium where high-type banks issue a credit derivative that guarantees the first-best level of profits*; *low-type banks offer a plain vanilla contract.*

Proof. See Appendix A. □

Remark 1. This separating equilibrium is the unique perfect Bayesian equilibrium satisfying the Cho–Kreps intuitive criterion.

The intuitive criterion proposed by Cho and Kreps (1987) for a signaling game (denoted "CK-criterion") is used to overcome the multiplicity of perfect Bayesian equilibria. The intuition behind this refinement concept is as follows. Consider a bank making an out-of-equilibrium proposal and any conjecture that the bank may have about the reaction of the risk-buyer. Given the risk-buyer's most optimistic conjecture (the risk-buyer believes that the proposer is a high-type bank, with probability one), if a high-type bank finds optimal to deviate while the low-type does not, then the intuitive criterion assigns probability one to the event that the proposer of such a contract is a high-type bank.

4.2.2. The optimal separating contract with costly capital

When bank capital is costly and capital requirements are based on maximal losses, the trivial contract $[P^{PS},\max(K-S,0)]$ is clearly no longer an optimal separating contract. In fact, it induces a capital requirement equal to (I_1+I_2) for the issuer of the security. The contract $[P^{NC}, L^{NC}]$ performs better since the induced capital requirement is equal to

$$R^{NC} = \left(L^{NC} + (I_1+I_2) - P^{NC}\right) = (I_1+I_2)\left(\frac{p_h + p_l - (1+\mu)p_h p_l}{p_h + p_l}\right), \tag{8}$$

which is positive but lower than (I_1+I_2). However, this is not the optimal contract since the amount of loss exposure, and therefore the amount of capital that a high-type bank needs to hold in order to signal its own type, should optimally depend upon the unitary cost of capital ρ. In fact, the larger the unitary cost of capital, the larger is the cost for the low-type bank in mimicking the high type. To find the optimal security, recall that both bank and risk-buyers are risk-neutral. Therefore the issuer, in order to minimize the maximum loss, splits up an equal repayment in all states of the world in which at least one default occurs. It is worth noticing that the default of one loan occurs with probability $2p_i(1-p_i)$ for a bank of type i. It follows from our assumption that, conditioned on the event "only one default" occurred, the probability that the bank is low-type is larger than the probability it is a high-type.[16]

Therefore, the optimal contract will be the separating contract that leaves the high-type bank the same payoff in the three default states. Observe that the optimal contract is different from the optimal contract in the DeMarzo and Duffie (1999) liquidity cost framework in which case bank's losses vary across default states.

Proposition 2. *When bank capital is costly, the optimal separating contract for high-type banks is the binary credit default basket contract $[P^C, L^C]$ such that*

$$L^C = \frac{(I_1+I_2)(1+\mu)(p_h(1+\rho) - p_l) - \rho(I_1+I_2)}{p_h^2(1+\rho) - p_l^2} \tag{9}$$

and

$$P^C = (1-p_h^2)\frac{(I_1+I_2)(1+\mu)(p_h(1+\rho) - p_l) - \rho(I_1+I_2)}{p_h^2(1+\rho) - p_l^2} + p_h(I_1+I_2)(1+\mu), \tag{10}$$

where P^C is the price paid by the risk-buyer for buying the bank portfolio and L^C is the amount that the bank has to pay if at least one loan defaults.

Proof. See Appendix A. □

Remark 2. The unique separating equilibrium that satisfies the CK-criterion is such that high-type banks offer the contract $[P^C, L^C]$ and low-type banks offer the plain vanilla contract. Risk-buyers' beliefs are such that a bank is high-type if and only if it offers the optimal separating contract $[P^C, L^C]$.

The maximum capital loss induced by the contract $[P^C, L^C]$ is equal to

$$\left(\frac{(p_h - p_l)((p_h+p_l)(I_1+I_2) - p_h p_l (I_1+I_2)(1+\mu))}{(p_h^2(1+\rho) - p_l^2)}\right) = R^C, \tag{11}$$

lower than R^{NC}.

Let $E\pi_h(P^C, L^C)$ denote the high-type bank's expected profit in signing the contract $[P^C, L^C]$. We then have:

$$E\pi_h\left(P^C, L^C\right) = (I_1+I_2)\left(p_h(1+\mu) - 1\right) - \rho R^C \tag{12}$$

i.e. the expected level of profits is equal to the first-best level minus the cost of capital.

[16] The event "one default" occurs with higher probability for a low-type than a high-type bank if the following holds: $p_h + p_l > 1$, which follows from our assumptions that the low-type NPV is positive and $\mu < 1$.

A high-type bank signals its type by committing to pay a fixed penalty in case at least one loan defaults. This signal turns out to be costly when bank capital itself is costly. However, the cost is lower than the cost a high-type bank faces signing the contract $[P^{NC}, L^{NC}]$. Not surprisingly, we have that $L^C = L^{NC}$ if $\rho = 0$, and $\frac{\partial R^C}{\partial \rho} < 0$, that is, the maximum capital loss is decreasing in the unitary cost of capital ρ. In fact, the maximum capital loss enters into the low-type bank's self-selection constraint, thereby reducing the incentive for the low-type bank to offer this type of contract. In particular, R^C tends to zero when the unitary cost of capital tends to infinity. The presence of capital requirements affects the level of high-type bank's profits. Moreover, the set of optimal contracts shrinks to include only those with a fixed amount repayment.

4.2.3. The optimal separating contract when capital is costly and the credit derivative market is opaque

The contract $[P^C, L^C]$ is separating only if the bank can to commit to bear the cost of capital ρR^C. From now on, we assume that the bank is not able to make such commitment. A contract in which a bank commits to retain this cost is not verifiable in front of a court, even if it may be observed by those who bought fractions of this security. It follows that the unitary cost of capital ρ cannot appear in the self-selection constraint for the low-type bank. In fact a low-type bank, after having sold the Binary CDB can buy a hedging contract at price $(1 - p_l^2)$ per unit of fixed amount in case of default and avoid the cost of capital ρR^C. The combination of the credit derivative contract $[P^C, L^C]$ and a contract hedging L^C provides larger expected profits to the low-type bank than hedging the loan portfolio with the CDS basket. Therefore, it provides incentives to mimic the high-type bank. The same reasoning holds when credit derivative contracts are private, and therefore the party who buys a portion of the security cannot even observe if the bank secretly hedges L^C with some other risk-buyers. Since trading in the Binary CDB market is relatively opaque, it appears that observability is the main relevant issue we are considering here.[17]

Lemma 1. *If the credit derivative market is opaque then the contract $[P^C, L^C]$ does not sustain a separating equilibrium.*

Proof. See Appendix A. □

The lack of commitment affects both the optimal separating contract and the level of profits of the high-quality banks.

Proposition 3. *When bank capital is costly and the credit derivative market is opaque, if $\rho \leqslant (p_h^2 - p_l^2)/p_l^2$, the optimal separating contract for high-type banks is the contract $[P^{NC}, L^{NC}]$, which is the optimal separating contract with fixed payment when capital is costless. If $\rho > (p_h^2 - p_l^2)/p_l^2$, then the optimal separating contract is*

$$P^O = \frac{(I_1 + I_2)(p_l(1+\mu) - 1 + p_l^2)}{p_l^2} \tag{13}$$

and

$$L^O = \frac{(I_1 + I_2)(p_l(1+\mu) - 1)}{p_l^2}. \tag{14}$$

Proof. See Appendix A. □

The intuition for this result is that there are two types of credit derivative contracts that can credibly sustain a separating contract. The first is the credit derivative contract $[P^{NC}, L^{NC}]$ when capital

[17] Opaqueness in trading affects CDS markets, while problems in pre-commitment to retaining risk are more related to CDO markets. Nevertheless, their effects in terms of contract design are the same.

is costless. As we argued above, this implies for the high-type bank a positive cost of capital equal to $\rho R^{NC} > 0$. The second type is the separating contract $[P^O, L^O]$, where the high-type bank faces no capital cost since

$$R^O = L^O + (I_1 + I_2) - P^O = 0. \tag{15}$$

The high-type bank's expected profits when it signs the contract $[P^O, L^O]$ are

$$E\pi_h\big(P^O, L^O\big) = p_h^2 \frac{(I_1 + I_2)(p_l(1+\mu) - 1)}{p_l^2}. \tag{16}$$

The contract $[P^O, L^O]$ is less profitable than the contracts $[P^C, L^C]$ and $[P^{NC}, L^{NC}]$ for the high-type bank: $E\pi_h(P^C, L^C) > E\pi_h(P^{NC}, L^{NC}) > E\pi_h(P^O, L^O)$. In fact, the contract $[P^O, L^O]$ is constrained by the non-negativity of the bank's profits in all the states (and especially in the worst state of the world). Thus, capital requirements are equal to zero. A high-type bank cannot use the signaling content implicit in the retention of some costly risk, i.e. a risk that generates a reduction of expected profits. Since the signal is less powerful, separation can only be obtained by reducing the price P at which the loans are sold, thereby reducing the incentive of the low-type bank in offering this type of contract. However, the contract $[P^{NC}, L^{NC}]$ sustains a separating equilibrium only if the high-type bank prefers to face the cost of capital, rather than hedging their capital loss as a low-type bank.

Let X denote the amount of coverage in case of default gained by a bank that buys a hedging contract at price $(1 - p_l^2)X$ to avoid the cost of capital. Then X is such that:

$$X - \big(1 - p_l^2\big)X - R^{NC} = 0, \tag{17}$$

that is,

$$X = \frac{R^{NC}}{p_l^2}. \tag{18}$$

The additional expected cost for a high-type bank that buys a hedging contract of amount X as a low-type bank is given by

$$X\big(1 - p_h^2\big) - X\big(1 - p_l^2\big) = -X\big(p_h^2 - p_l^2\big), \tag{19}$$

so that the contract $[P^{NC}, L^{NC}]$ is a separating contract only if the expected cost of hedging the loss as a low-type bank is lower than or equal to the expected cost of capital, that is

$$X\big(p_h^2 - p_l^2\big) \leqslant \rho R^{NC}. \tag{20}$$

Substituting (18) in (20) and dividing by the maximum capital loss R^{NC} we have:

$$\rho \geqslant \frac{(p_h^2 - p_l^2)}{p_l^2}. \tag{21}$$

When the bank capital is costly and the credit derivative market is opaque, we have two cases. If $\rho \leqslant (p_h^2 - p_l^2)/p_l^2$ then the unique CK-separating equilibrium has the high-type bank offering the contract $[P^{NC}, L^{NC}]$ and the low-type bank offering a plain vanilla contract at their fair price. Risk-buyers' beliefs are such that a bank is high-type if and only if it offers the contract $[P^{NC}, L^{NC}]$. If $\rho > (p_h^2 - p_l^2)/p_l^2$, then the unique CK-separating equilibrium is such that the high-type bank offers the contract $[P^O, L^O]$ and the low-type bank offers a plain vanilla contract at their fair price. Risk-buyers' beliefs are such that a bank is high-type if and only if it offers the contract $[P^O, L^O]$.

The level of profits that a high-type bank gets by signing the $[P^O, L^O]$ contract is not only lower than the first best level of profits, but it can be even lower than the level of profits the high-type gets in the pooling equilibrium.

Lemma 2. *If $\rho > (p_h^2 - p_l^2)/p_l^2$, $q > (p_h^2(\frac{(1+\mu)}{p_l} - p_l^2) - \mu - (1 - p_l)(1+\mu))/((p_h - p_l)(1+\mu))$ and the credit derivative market is opaque, then the high-type banks' profits are larger in the pooling equilibrium than in the optimal separating equilibrium.*

Proof. See Appendix A. □

4.3. Informed regulators

Until now we have assumed that there exists asymmetric information between the bank and the risk-buyer, and we have not mentioned what kind of information regulators own. Recently, there have been debates over what kind of information should be disclosed by the regulators, who are usually better informed than the market, in order to reduce problems of asymmetric information (see Morrison, 2005) without harming banks' strategies and without favoring collusive behavior in the credit markets. We present a simple example describing how regulators can implicitly inform the market without disclosing any sensitive information by imposing different capital requirements to different bank-types. Suppose that the regulatory Authority knows the bank types and $(1-p_h)^2 < 1-\alpha < (1-p_l)^2 \leqslant 1-p_h$. If a high-type bank holds the loans in its portfolio its capital requirement is equal to $\max\{I_1, I_2\}$, while a low-type bank has a capital requirement equal to $I_1 + I_2$. A high-type bank can now design a simple contract which sets at zero its capital requirement and signals its quality to the market. Moreover, the optimal separating contract for high-type banks guarantees the first-best level of profits, even if the market is opaque.

This contract, which is one of the optimal contracts in case of costless capital, concentrates all the losses in the state of the world that does not generate capital requirements for the high type, as stated in the following proposition.

Proposition 4. *When $(1-p_h)^2 < 1-\alpha < (1-p_l)^2$ and regulators are informed on the banks' type, the optimal contract requires that the high-type bank sells its loans at price*

$$P^{RI} = (1-p_l)^2 \frac{(I_1+I_2)(1+\mu)}{2-p_h-p_l} + p_l(I_1+I_2)(1+\mu) \tag{22}$$

and commits to pay an amount

$$L^{RI} = \frac{(I_1+I_2)(1+\mu)}{2-p_h-p_l} \tag{23}$$

to the risk-buyer if and only if both loans default.

Proof. See Appendix A. □

The intuition behind this result is straightforward. Under the assumption that the informed regulator does not impose capital requirement for losses that occur with a probability lower than α, the high-type bank exploits capital requirements to signal its type.

4.4. A simple extension to the continuous case

Up to now, we considered a simple discrete case where the only publicly observable signal is the default of the loans in the portfolio. In this section we consider a more general model where the initial investment of the bank on the loan portfolio is I, and the public observable signal is the realized cash flows of the loan portfolio, which is assumed to be a continuous random variable $s \in [0, M]$, where M is the maximum cash flow generated when all loans are repaid. Hence, we model the security as a continuous and differentiable function $g(s) : [0, M] \to \mathbb{R}_+$ which specifies the non-negative amount that the bank pays to the risk-buyer for any realization of the cash flow s of the loan portfolio. Let $\bar{s} \in \arg\max g(s)$, that is $g(\bar{s})$ is the maximum amount that the bank agrees to pay to the risk-buyer,[18] which is relevant to the determination of capital requirements. Let ρ indicate, as usual, the unitary cost of capital due to the presence of capital requirements and $F_i(s)$ $(f_i(s))$ denote the conditional distribution (density) function of s given type $i = h, l$. Let $E_i(s)$ be the expected cash-flow of the loan

[18] Here we do not discuss the role of informed regulators, and we extend the confidence level of capital requirements to 100%.

portfolio if the bank is of type i. A contract is a pair $[P, g(s)]$, where P is the price paid to the bank. Consider directly the case where the credit derivative market is opaque. A high-type bank that offers a separating contract, solves the following problem:

$$\max_{\{g(s),P\}} P - I - \int_0^M g(s) f_h(s)\,\mathrm{d}s + \rho\big(\min\{0, P - I - g(\bar{s})\}\big) \tag{24}$$

s.t.

$$\int_0^M g(s) f_h(s)\,\mathrm{d}s + \int_0^M s f_h(s)\,\mathrm{d}s - P \geqslant 0, \tag{25}$$

$$P - \int_0^M g(s) f_l(s)\,\mathrm{d}s \leqslant \int_0^M s f_l(s)\,\mathrm{d}s, \tag{26}$$

$$\bar{s} \in \arg\max g(s). \tag{27}$$

Note that the cost of capital does appear in the incentive compatibility constraint, because the low-type bank can secretly hedge any portion of the security. The solution to the problem depends upon the specification of the conditional density functions $f_i(s)$ for $i = h, l$. We provide a sufficient condition that leads to a result similar to the discrete case. Namely, we assume that there exists $s_0 \in [0, M]$ such that $0 < f_h(s) \leqslant f_l(s)$ for all $s \leqslant s_0$ and $f_h(s) > f_l(s) > 0$ for all $s > s_0$.

Proposition 5. *If there exists a separating equilibrium, then the optimal separating contract for the high-type banks is a pair* $(P, g(s))$ *where*: $g(s) = \frac{E_h(s) - E_l(s)}{F_l(s_0) - F_h(s_0)} = k$ *for all* $s \leqslant s_0$ *and* $g(s) = 0$ *for all* $s > s_0$ *and* $P = \frac{E_h(s) F_l(s_0) - E_l(s) F_h(s_0)}{F_l(s_0) - F_h(s_0)}$.

Proof. See Appendix A. □

Proposition 5 states that the optimal contract under the continuous distribution of loan payoffs is still a Binary CDB where, if the realization of loan repayments is below s_0, the bank pays a fixed amount k. This is in line with the optimal contract in the discrete payoff case.

As stressed before, this optimal contract is different than the optimal contract in the DeMarzo and Duffie (1999) liquidity cost framework. It is easy to observe that this result is not driven by the unlimited liability assumption we made differently from DeMarzo and Duffie (1999), but from capital requirements based on maximum losses. Without unlimited liability, in the DeMarzo and Duffie (1999) framework the optimal contract would be the trivial one in which a large penalty is attached to the state with the largest difference in the likelihood of being a low and a high type. Because of the capital requirement rule based on maximal losses, this is clearly not the optimal contract in our framework.

5. Regulatory and credit market implications

The optimal contracts described above fit well with many of the contracts we observe in the market. The contract $[P^{PS}, \max(K - S, 0)]$ is similar to a CDO where the bank retains the equity tranche in order to signal its type. The $[P, L]$ is similar to a contract where the bank sells the loan portfolio and commits to pay a fixed amount if losses are above a certain level, i.e. it writes a Binary CDB. It is important to stress that other combinations of contracts that lead to equal exposure across states could generate the same payoffs as the $[P, L]$ contracts.[19]

[19] An example is provided in Nicolò and Pelizzon (2005). When loans have different maturities, the bank commits to provide a credit enhancement if future losses are large. Indeed, purchasing short term protection with a first-to-default contract, and

We investigate whether the different institutional settings may influence the structure of optimal signaling contracts and therefore the demand of different credit derivatives for capital requirements reduction.[20]

In the last two decades we have observed an attempt to change bank capital regulations. In the early 1980s, as concerns for the financial health of international banks mounted and complaints of unfair competition increased, the Basel Committee on Banking Supervision initiated a discussion on the revision of capital standards. An agreement was reached in July 1988. The 1988 Basel Accord (Basel I, BCBS, 1988) explicitly considered only credit risk and imposed an 8% capital requirement on risk-adjusted assets.[21] The Accord does not recognize credit derivatives explicitly. The basic approach taken by regulators in official announcements is to draw analogies with more conventional instruments for which a well-developed regulatory framework already exists.[22]

One aspect that needs to be considered is that, under Basel I, in most jurisdictions CDOs are considered as a portfolio of loan sales and banks face almost no capital requirements for holding the equity tranche. This setting is similar to our case where the required capital for holding the equity tranche has a cost equal to zero. Indeed, this implies that under Basel I, high-quality banks can optimally use the equity tranche holding to signal their own type.

However, following its introduction, the Accord has been fine-tuned to accommodate financial innovation and to reflect some risks not initially considered. In particular, (see BCBS, 2005) a new approach, Basel II, has recently been proposed to take into account some of the limitations of the earlier framework. More specifically, the introduction of Basel II capital adequacy rules requires for CDOs that all first-loss positions must be deducted from bank capital.[23] This requirement is similar to our framework with costly capital. Our results indicate that the introduction of such a capital requirement may prevent the use of the equity tranche holding, and other contracts characterized by loss smoothing will be used as signaling devices. An example of such contracts is the Binary CDB. However, the optimality of such a contract under Basel II capital adequacy rules could be compromised by the opacity in the OTC credit derivative markets. In fact, our model suggests that when banks cannot commit to terminal-date risk exposure due to the opacity of the market, either a high-type bank holds a larger fraction of risk or, in case the cost of capital is too high, it signals its type with its initial-date price, i.e. through underpricing.

Does experience provide support for these arguments, or are these changes too new for us to know? The answer to this question is not easy. It is worth stressing that, though Basel II has been operating since January 2007, most countries have set January 2008 as the start date for the adjustment to the new accord. Consequently, these recent changes are too recent to offer strong empirical evidence. However, there are already signs in the market that banks have started to sell first-loss pieces in order to avoid the onerous dollar-for-dollar capital deductions under new regulations (see Marray, 2006 and Duffie, 2007). Preliminary evidence is also provided by J.P. Morgan that in 1999 issued the BISTRO (Broad Index Secured Trust Offering) product to lower its own capital commitment to credit risk. The BISTRO is characterized by the fact that if there is default beyond some level x, then the CDO issuer will cover the losses. Note that the threshold x is fixed so that the risk of the loss exceeding x is, at most, remote (better than AAA rating), implying a very limited maximal loss.[24] This product is similar in spirit to the optimal separating contract described in Section 4.3, where the

committing to buy insurance in the future in case of defaults, is perfectly in line with the $[P, L]$ contract (the Binary CDB contract). Analytically, assuming that I_1 loan has a shorter maturity than the I_2 loan a first-to-default contract plus a commitment to buy insurance in case of any loan default induces the same payoffs as the Binary CDB. The proof is available from the authors upon request.

[20] Note that credit derivatives' demand is related not only to capital requirements but also to liquidity problems. For an overview and an empirical analysis of why banks use credit derivatives, see Minton et al. (2006).

[21] Risk-adjusted assets are defined as a weighted sum of bank assets whose weights depend on asset risk buckets. See Cooke (1990) for the debate that leads to the Accord, BCBS (2005) for the Accord and its amendments, and Pelizzon and Schaefer (2006) for a detailed description of the main advantages and drawbacks of the new Accord.

[22] For an overview of regulatory announcements, see Staehle and Cumming (1999).

[23] See BCBS (2005, p. 48).

[24] For a more specific description of a BISTRO contract, see JP Morgan (2006).

CDO issuer commits to pay losses if they are above a certain level. This payment is designed to be as low as possible in terms of capital requirements (like a AAA product).

More recently, we have started to observe one of the most relevant security design innovation in credit market: the creation of credit derivative product companies (CDPCs). CDPCs are special-purpose structured finance operating companies whose only permitted line of business is to sell credit protection (like BISTRO SPV). The key feature of these companies is the capital structures designed for AAA ratings in order to take advantage of the opportunity to sell protection without posting collateral on their portfolio of loans. This is reached through strict contractual risk limits that, in case of breach, force either an immediate liquidation or a freezing of the CDPC portfolio. Rating Agencies certify not only that such companies are indeed able to commit in retaining a fraction of risk but also that they are able to limit the risks that they are retaining through diversification and risk management. As suggested by Duffie (2007) "the CPDC can serve as a flexible and ongoing financing conduit for a sponsor with a pipeline of loan risk." Since CPDCs are more transparent than banks, rating agencies may have less difficulties to evaluate the risk of these specialty companies and to provide AAA ratings. Nevertheless, it is not clear whether rating agencies are always able to evaluate CDPC portfolio risk. In fact, the recent sub-prime crisis indicates that this may not always be the case.

Our model also suggests that opacity may generate underpricing of credit derivative products when capital requirements make the retention of risk too costly for the bank. As stressed above, under Basel I the issue of CDO as credit risk transfer does not imply underpricing, as ex ante commitment (holding the first loss) for risk taking was not costly. However, for other credit risk transfer instruments subject to large capital requirements under Basel I (like CDS) we may expect underpricing. The aim of our work is not to investigate this issue empirically. However, Ashcraft and Santos (2007) find that "[d]espite widespread claims by economists and policymakers about the benefits of credit derivatives, the average corporate borrower has not yet seen a lower cost of debt capital. More interestingly, the types of firms that one expects would naturally benefit the most—risky and informationally opaque firms—appear to have been adversely affected by the CDS market." This potentially comes from the lemons' premia that banks need to pay to risk-buyers because of their inside information regarding credit risk. These lemons' premia may then be transferred to borrowers. The evidence is the same for loan sales as stressed by Duffie (2007): "... banks indeed have private information about a borrower's default risk, and that banks are likely to suffer lemon's premia from loan sales, are consistent with research by Dahiya, Puri, and Saunders (2003) and Marsh (2006) who show that sale of a bank loan is associated with a significant drop in the price of the borrower's equity."[25] Those are only preliminary evidences. Whether opacity generates underpricing of CDS products, and whether the introduction of Basel II in presence of opacity generates underpricing also for CDO, are issues that need further investigation.

6. Concluding remarks

In this paper we show that optimal security design may change depending on regulatory capital requirements and on the presence of opacity in credit derivative markets. The opaqueness of credit derivative markets appears to be an important issue that may deserve further investigation, as highlighted by the recent sub-prime crisis. We propose three extensions of the present paper. First, we have considered only frictions that come from capital adequacy rules in presence of a positive cost of capital. Liquidity costs may also provide frictions that induce credit risk transfer. Thus, it is worth investigating how the joint effects of these frictions influence the optimal contract design. Second, we have assumed that loans in the portfolio are equally risky. It would be interesting to analyze how the optimal contract design changes when the characteristics of the asset pools vary. Third, we suggest to investigate how partial reforms of the institutional framework can help to mitigate the problem of opacity in the credit derivative markets.

[25] In our model we focus on the adverse selection problem. However, a similar argument can be provided for the moral hazard problem that arises from the disincentive to monitor when banks transfer the credit risk of their loans.

Appendix A

Proof of Proposition 1. We show that there exists a separating contract $[P, F(S)]$ that provides first-best level of profits to high-type banks. We consider first the case where $F(S) = \eta(\max(K - S, 0))$ with $K = (I_1 + I_2)(1 + \mu)$. In such a contract a bank sells its loans I_1 and I_2 to the risk-buyer at price P and commits to cover a fraction η of the risk-buyer's losses in case of default of any loan. A separating equilibrium has to satisfy both the participation constraint for the risk-buyer and the self-selection constraint for the low-type, that is its expected profits are not larger when it offers the $[P, \eta(\max(K - S, 0))]$ contract than when it offers a CDS basket at its fair price.

Namely, the optimal separating contract $[P^{PS}, \eta(\max(K - S, 0))]$ solves the following problem:

$$\max_{\{P,\eta\}} P - (1 - p_h)p_h\eta(I_1 + I_2)(1 + \mu) - (1 - p_h)^2\eta(I_1 + I_2)(1 + \mu) \tag{28}$$

s.t.

$$P - \eta(I_1 + I_2)(1 + \mu)\big((1 - p_l)p_l + (1 - p_l)^2\big) \leqslant p_l(I_1 + I_2)(1 + \mu), \tag{29}$$

$$\eta(I_1 + I_2)(1 + \mu)(1 - p_h) + p_h(I_1 + I_2)(1 + \mu) - P \geqslant 0. \tag{30}$$

Given the linearity of the problem, the solution can be easily obtained by solving the two constraints as equality and we obtain $\eta = 1$ and $P^{PS} = (I_1 + I_2)(\mu + 1)$; the high-type banks' expected profits are equal to $E\pi_h = (I_1 + I_2)p_h(1 + \mu)$, which clearly are the first-best level of profits for high-type banks. The separating equilibrium that satisfies the CK-criterion is the strategy profile in which high-type banks offer the $[P^{PS}, \max(K - S, 0)]$ contract above described and low-type banks offer full-coverage plain vanilla contracts at the fair price $(1 - p_l)(I_1 + I_2)$. Risk-buyer beliefs are such that a bank is high-type with probability one if and only if it offers the $[(I_1 + I_2)(1 + \mu), \max(K - S, 0)]$ contract. It is easy to check that these beliefs satisfy the intuitive criterion. To check for uniqueness (in welfare terms), consider a separating equilibrium such that high-type banks sign a contract $[P', \eta']$ and make profits that are lower than the first-best profit levels. There always exist a contract $[P'', \eta'']$ with $P'' > P'$ and $\eta'' > \eta'$, that, if offered by a high-type bank, leads to first-best profit levels, while, if offered by a low-type bank, leads to profits that are lower than $(I_1 + I_2)p_l(1 + \mu)$.

Note that there exist other separating contracts that yield the same expected profits for the high-type bank. For example, the contract $[P, L]$ where L is a flat refund payment in case of any default (i.e. $F(S) = L | S < (I_1 + I_2)(1 + \mu)$) and P is the price offered for the two loans (a Binary CDB contract).

The maximum price that the counterpart agrees to pay is equal to the expected profits of the loans minus the expected payment received in case of default of any of the underlying assets. The optimal separating contract $[P^{NC}, L^{NC}]$ satisfies the following problem:

$$\max_{\{P,L\}} P - \big(1 - p_h^2\big)L \tag{31}$$

s.t.

$$P - \big(1 - p_l^2\big)L \leqslant p_l(I_1 + I_2)(1 + \mu), \tag{32}$$

$$p_h(I_1 + I_2)(1 + \mu) + \big(1 - p_h^2\big)L - P \geqslant 0. \tag{33}$$

The solution of the problem is straightforward since it satisfies the above constraints as equality:

$$L^{NC} = \frac{(I_1 + I_2)(1 + \mu)}{(p_h + p_l)}, \tag{34}$$

$$P^{NC} = (I_1 + I_2)(1 + \mu)\left(\frac{1 + p_h p_l}{p_h + p_l}\right) \tag{35}$$

and it provides the same expected payoff to the high-type bank as the contract $[P^{PS}, \max(K - S, 0)]$, i.e. $E\pi_h = p_h(I_1 + I_2)(\mu + 1)$. Therefore, it is an optimal separating contract.

More specifically, it follows that there exists a separating $[P^{NC}, L^{NC}]$ contract such that high-type banks sell loans I_1 and I_2 in exchange for an amount of money equal to P^{NC} and commit to pay L^{NC} in case of the default of any loan. Low-type banks sign full-coverage plain vanilla contracts paying the fair premium $(1 - p_l)(I_1 + I_2)(1 + \mu)$. The equilibrium beliefs are such that the risk-buyer believes that the bank is high-type if and only if it offers to sign a $[P^{NC}, L^{NC}]$ contract. It is easy to check that these beliefs satisfy the intuitive criterion. □

Proof of Proposition 2. We first prove that the optimal separating contract provides for a flat repayment in case any default occurs. We denote by L_1 the amount of repayment in case only one default occurs and by L_2 the amount of the repayment in case both assets default. The optimal separating contract $[P, L_1, L_2]$ solves the following problem:

$$\max_{\{P,L_1,L_2\}} P - (1 - p_h)^2 L_2 - 2p_h(1 - p_h)L_1 - \rho\big(\max 0, L_2 + (I_1 + I_2) - P\big) - (I_1 + I_2) \tag{36}$$

s.t.

$$P - (1 - p_l)^2 L_2 - 2p_l(1 - p_l)L_1 - \rho\big(\max 0, L_2 + (I_1 + I_2) - P\big)$$
$$\leqslant p_l(I_1 + I_2)(1 + \mu), \tag{37}$$

$$(1 - p_h)^2 L_2 + 2p_h(1 - p_h)L_1 - P + p_h(I_1 + I_2)(1 + \mu) \geqslant 0, \tag{38}$$

$$L_2 - L_1 \geqslant 0, \tag{39}$$

$$L_1 \geqslant 0, \tag{40}$$

First-order conditions are:

$$1 + \rho - \lambda(1 + \rho) - \mu = 0, \tag{41}$$

$$\delta - (1 - p_h)^2 - \rho + \lambda\big(1 - p_l^2\big) + \lambda\rho + \mu(1 - p_h)^2 = 0, \tag{42}$$

$$\gamma - 2p_h(1 - p_h) + \lambda 2p_l(1 - p_l) + \mu 2p_h(1 - p_h) - \delta = 0. \tag{43}$$

Suppose first that $\delta = 0$ (and therefore $L_2 > L_1$). From (41)

$$\mu = (1 + \rho)(1 - \lambda). \tag{44}$$

Substituting in (42) we obtain

$$\lambda = \frac{1}{(1 + \rho)(1 - (1 - p_h)^2) - p_l^2}\rho\big(1 - (1 - p_h)^2\big) \tag{45}$$

which implies $\lambda < 1$. Substituting (45) and (44) in (43) we obtain

$$\gamma = 2\rho p_h(1 - p_h)(\lambda - 1) - 2\lambda(p_h - p_l)(p_h + p_l - 1). \tag{46}$$

Since $\lambda < 1$, the first term is negative. If $p_h + p_l > 1$ the second term is also negative, reaching the desired contradiction. We assumed that the NPV is positive for both types ($p_l > \frac{1}{1+\mu}$) and that $\mu < 1$. These are sufficient conditions for $p_h + p_l > 1$. In the same way we reach a contradiction when $L_1 > L_2$ is assumed. Hence, to find the optimal separating contract we simply solve the following system of equations:

$$P - \big(1 - p_l^2\big)L - \rho\big(\max 0, L + (I_1 + I_2) - P\big) - p_l(I_1 + I_2)(1 + \mu) = 0, \tag{47}$$

$$\big(1 - p_h^2\big)L - P + p_h(I_1 + I_2)(1 + \mu) = 0. \tag{48}$$

It follows that the optimal separating contract $[P^C, L^C]$ is such that

$$L^C = \frac{(I_1 + I_2)(1 + \mu)(p_h(1 + \rho) - p_l) - \rho(I_1 + I_2)}{(p_h^2(1 + \rho) - p_l^2)} \tag{49}$$

and

$$P^C = \left(1 - p_h^2\right)\frac{(I_1 + I_2)(1+\mu)(p_h(1+\rho) - p_l) - \rho(I_1 + I_2)}{(p_h^2(1+\rho) - p_l^2)} + p_h(I_1 + I_2)(1+\mu). \tag{50}$$

Note that the optimal separating contract $[P^C, L^C]$ implies that the bank has a negative payoff in the worst state of the world (that is when both loans default), in fact

$$P^C - L^C - (I_1 + I_2) = (I_1 + I_2)\left((1+\mu)p_h p_l - (p_h + p_l)\right) < 0 \tag{51}$$

if and only if $\mu < \frac{p_h + p_l - p_h p_l}{p_h p_l}$, which holds by the assumption that $\mu < 1$. Therefore, any contract for which the above constraint is satisfied as equality induces a positive cost of capital.

It is easy to check that indeed the high-type bank has a non-zero capital requirement, since $P^C - L^C - (I_1 + I_2) < 0$. Moreover, $L^C = L$ if $\rho = 0$ and $\frac{\partial L^C}{\partial \rho} < 0$. Let $R^C \equiv L^C + (I_1 + I_2) - P^C$ denote the capital requirement. We have that

$$R^C = \frac{(p_h - p_l)((p_h + p_l)(I_1 + I_2) - p_h p_l (I_1 + I_2)(1+\mu))}{(p_h^2(1+\rho) - p_l^2)} \tag{52}$$

and therefore $\lim_{\rho\to\infty} R^C = 0$ and $\lim_{\rho\to\infty} L^C = \frac{(I_1+I_2)(p_h(1+\mu)-1)}{p_h^2}$.

Let $E\pi_h(P^C, L^C)$ denote the high-type expected profit in signing the contract $[P^C, L^C]$. We have

$$E\pi_h\left(P^C, L^C\right) = (I_1 + I_2)\left(p_h(1+\mu) - 1\right) - \rho R^C. \tag{53}$$

The expected profit is equal to the first-best profit minus the signaling cost ρR^C. This derives from the fact that the high-type bank signals its type by paying a penalty if at least one of the loans defaults (thus covering part of the losses of the risk-buyer in case of default). Note that the capital loss is decreasing in the cost of capital ρ since it enters negatively in the low-type bank constraint and therefore reduces the incentive of the low-type bank to sign this type of contract. The separating equilibrium that satisfies the CK-criterion is the strategy profile in which high-type banks offer the contract $[P^C, L^C]$ and low-type banks offer full-coverage plain vanilla contracts at the fair price $(1 - p_l)(I_1 + I_2)$. Risk-buyer beliefs are such that a bank is high-type with probability one if it offers the contract $[P^C, L^C]$; and a low-type with probability one otherwise. The same argument presented in the proof of Proposition 1 proves that this contract is the unique one satisfying the CK-criterion. □

Proof of Lemma 1. In any separating equilibrium the expected profit of the low-type bank is

$$E\pi_l = (I_1 + I_2)\left(p_l(1+\mu) - 1\right). \tag{54}$$

If the low-type bank deviates, signs the contract $[P^C, L^C]$, and hedges the repayment L^C by means of a hedging contract bought at price $(1 - p_l^2)L^C$, it obtains

$$E\pi_l\left(P^C, L^C\right) = (I_1 + I_2)\left(p_h(1+\mu) - 1\right) - \left(p_h^2 - p_l^2\right)L^C. \tag{55}$$

An easy calculation shows that $E\pi_l(P^C, L^C) > E\pi_l$ if and only if

$$\mu < \frac{p_h + p_l - p_l p_h}{p_l p_h} \tag{56}$$

that is true by the assumption that $\mu < 1$. □

Proof of Proposition 3. If the credit derivative market is opaque, the risk seller can secretly hedge its risk exposure with the Binary CDB contract without the risk-buyer being informed.

In this case, the optimal separating contract $[P^{NC}, L^{NC}]$ solves the following problem:

$$\max_{\{P,L\}} P - \left(1 - p_h^2\right)L + \rho\left(\min\left\{0, P - L - (I_1 + I_2)\right\}\right) - (I_1 + I_2) \tag{57}$$

s.t.

$$P - \left(1 - p_l^2\right)L \leqslant p_l(I_1 + I_2)(1 + \mu), \tag{58}$$

$$p_h(I_1 + I_2)(1 + \mu) + \left(1 - p_h^2\right)L - P \geqslant 0. \tag{59}$$

Note that the cost of capital does not enter in the low-type bank's self selection constraints since a low-type can always fully hedge the payment of the fixed amount L at its fair price $(1 - p_l^2)L$. Hence, the optimal contract when capital is costless $[P^{NC}, L^{NC}]$ is also the natural candidate to be the optimal separating contract. Consider the strategy profile such that high-type banks sign contracts $[P^{NC}, L^{NC}]$ and low-type banks sign CDS basket contracts. The only system of beliefs consistent with this strategy profile is such that the risk-buyer assigns probability one to the bank being high-type if and only if it offers the contract $[P^{NC}, L^{NC}]$. The high-type bank expected profits, if it signs the contract $[P^{NC}, L^{NC}]$, are equal to

$$E\pi_h\left(P^{NC}, L^{NC}\right) = (I_1 + I_2)\left(p_h(1 + \mu) - 1\right) - \rho R^{NC} \tag{60}$$

where $\rho R^{NC} = \rho((I_1 + I_2)\frac{((p_h+p_l)-(1+\mu)p_h p_l)}{(p_h+p_l)})$ is the total cost of capital, as before. The separating equilibrium exists only if high-type banks do not deviate and buy CDS basket contracts as low-types. The optimal deviation for the high-type bank is buying a hedging contract of amount X at a unit price of $(1 - p_l^2)$, such that

$$X - \left(1 - p_l^2\right)X - R^{NC} = 0, \tag{61}$$

that is,

$$X = \frac{R^{NC}}{p_l^2}. \tag{62}$$

Let $E\pi_h(P^{NC}, L^{NC}, X)$ denote the expected profits of the high-type bank when it signs a contract $[P^{NC}, L^{NC}]$, and a plain vanilla contract of amount X as a low-type. Then, we have that

$$E\pi_h\left(P^{NC}, L^{NC}, X\right) = (I_1 + I_2)\left(p_h(1 + \mu) - 1\right) - \left(p_h^2 - p_l^2\right)X \tag{63}$$

and $E\pi_h(P^{NC}, L^{NC}) \geqslant E\pi_h(P^{NC}, L^{NC}, X)$ if and only if

$$\rho \leqslant \frac{p_h^2 - p_l^2}{p_l^2}. \tag{64}$$

If $\rho > (p_h^2 - p_l^2)/p_l^2$ then the contract $[P^{NC}, L^{NC}]$ does not sustain a separating equilibrium. In fact, if $\rho > (p_h^2 - p_l^2)/p_l^2$, a high-type prefers to buy protection as a low type than sustaining the cost of capital. The separating equilibrium exists only if there is no capital requirement for the bank loan portfolio. Therefore the optimal separating contract $[P^O, L^O]$ satisfies the following problem:

$$\max_{\{P,L\}} P - \left(1 - p_h^2\right)L - (I_1 + I_2) \tag{65}$$

s.t.

$$P - \left(1 - p_l^2\right)L - p_l(I_1 + I_2)(1 + \mu) \leqslant 0, \tag{66}$$

$$p_h(I_1 + I_2)(1 + \mu) + \left(1 - p_h^2\right)L - P \geqslant 0, \tag{67}$$

$$P - (I_1 + I_2) - L \geqslant 0. \tag{68}$$

The solution of this maximization problem is the following:

$$P^O = \frac{(I_1 + I_2)(p_l(1 + \mu) - 1 + p_l^2)}{p_l^2}, \tag{69}$$

$$L^O = \frac{(I_1 + I_2)(p_l(1 + \mu) - 1)}{p_l^2}. \tag{70}$$

A further constraint is added in the case of no capital requirements. The effect of capital requirements and opacity turn out to be similar to a liquidity constraint such that in no state of the world the bank has to face negative cash-flows. The high-type bank profits are equal to $E\pi_h(P^O, L^O) = \frac{p_h^2}{p_l^2}(I_1 + I_2)(p_l(1+\mu) - 1)$. □

Proof of Lemma 2. By simply comparing the two levels of profits we have that the level of profits of the pooling equilibrium is higher if

$$(I_1 + I_2) - \big(q(1-p_h) + (1-q)(1-p_l)\big)(I_1 + I_2)(1+\mu) \geqslant p_h^2 \frac{(I_1 + I_2)(p_l(1+\mu) - 1)}{p_l^2}, \tag{71}$$

that is,

$$q \geqslant \frac{p_h^2(\frac{(1+\mu)}{p_l} - p_l^2) - \mu - (1-p_l)(1+\mu)}{(p_h - p_l)(1+\mu)}. \qquad \square \tag{72}$$

Proof of Proposition 4. When $(1-p_h)^2 < 1-\alpha$ the high type bank has a capital requirement equal to $\max(I_1, I_2)$. If $(1-p_l)^2 > 1-\alpha$ then the low type bank has a capital requirement equal to $I_1 + I_2$. In this case the high type bank could exploit the asymmetry in terms of capital requirements in order to signal its type. The optimal separating contract $[P^{\mathrm{RI}}, L^{\mathrm{RI}}]$ satisfies the following problem:

$$\max_{\{P,L\}} P - (1-p_h)^2 L \tag{73}$$

s.t.

$$P - (1-p_l)^2 L \leqslant p_l(I_1 + I_2)(1+\mu), \tag{74}$$

$$p_h(I_1 + I_2)(1+\mu) + (1-p_h)^2 L - P \geqslant 0. \tag{75}$$

The solution of the problem is straightforward since it satisfies the above constraints as an equality:

$$P^{RI} = (1-p_l)^2 \frac{(I_1 + I_2)(1+\mu)}{2 - p_h - p_l} + p_l(I_1 + I_2)(1+\mu), \tag{76}$$

$$L^{RI} = \frac{(I_1 + I_2)(1+\mu)}{2 - p_h - p_l}. \tag{77}$$

With this optimal contract the high-type bank sells its loans at price P^{RI} and commits to pay an amount L^{RI} to the risk-buyer if and only if both loans default. □

Proof of Proposition 5. First, suppose that there exists an optimal separating contract $(\tilde{P}, \tilde{g}(s))$ such that the function $\tilde{g}(s)$ is positive in a neighborhood of $s' > s_0$, i.e. $\tilde{g}(s) > 0$ for all $s \in [s' - \varepsilon, s' + \varepsilon]$. Let $\tilde{s} \in \arg\max \tilde{g}(s)$. Consider a function $g(s)$ such that $\int_0^{s_0} g(s) f_l(s)\,\mathrm{d}s = \int_0^{s_0} \tilde{g}(s) f_l(s)\,\mathrm{d}s$ and $g(s) = 0$ for $s > s_0$ and a price

$$P = \min\left\{\int_0^{s_0} g(s) f_h(s)\,\mathrm{d}s + E_h(s) + \rho R^S, E_l(s) + \int_0^{s_0} g(s) f_l(s)\,\mathrm{d}s + \rho R^S\right\} \tag{78}$$

where $R^S = \min\{0, P - I - g(\bar{s})\}$.

If the contract $[\tilde{P}, \tilde{g}(s)]$ is separating, then the contract $[P, g(s)]$ is separating too. Moreover $\tilde{g}(\tilde{s}) \geqslant g(\bar{s})$ and the contract $[P, g(s)]$ provides to the high-type banks at least the same level of profits.

Second, suppose that there exists an optimal contract $[\tilde{P}, \tilde{g}(s)]$ where $\tilde{g}(s) = 0$ for all $s \geqslant s_0$ and there exist $s', s'' < s_0$ with $\tilde{g}(s') > \tilde{g}(s'')$. Let $\tilde{s} \in \arg\max \tilde{g}(s)$. Let $E_i(s)$ denote the conditional expected value of the asset owned by type $i = h, l$. Since the contract $[\tilde{P}, \tilde{g}(s)]$ is separating, then high-type profits are at most $E_h(s) - I + \rho(\min\{0, P - I - g(\bar{s})\})$. Consider now a continuous function $g(s) = k$

(where k is a constant and does not depend on s) for all $s \leqslant s_0$ (and $g(s) = 0$ otherwise) and a contract $[P, k]$ such that

$$P = kF_h(s_0) + E_h(s), \tag{79}$$

$$P = kF_l(s_0) + E_l(s). \tag{80}$$

By construction, the contract $[P, k]$ is separating and it follows that $\tilde{g}(\tilde{s}) > k$. Therefore, high-type banks' profits are larger or equal, by signing a contract $[P, k]$, than by signing a contract $[\tilde{P}, \tilde{g}(s)]$. Solving the above equations we obtain

$$k = \frac{E_h(s) - E_l(s)}{F_l(s_0) - F_h(s_0)} \tag{81}$$

and

$$P = \frac{E_h(s)F_l(s_0) - E_l(s)F_h(s_0)}{F_l(s_0) - F_h(s_0)}. \qquad \square \tag{82}$$

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